Towards a universal gateset for QMA_1

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Highlights

1. $\forall k \in \mathbb{Z}_{\geq 0}$: QMA_1 in the *cyclotomic field* $\mathbb{Q}(\zeta_{2^k})$, $\zeta_m = e^{2\pi i/m}$ has a *universal gateset* $\mathcal{G}_{2^k}$.
   - Let $\mathcal{G} \subseteq \mathrm{U}(2^\ell, \mathbb{Q}(\zeta_{2^k}))$ be a finite gateset. Then $\mathsf{QMA}_1^{\mathcal{G}} \subseteq \mathsf{QMA}_1^{\mathcal{G}_{2^k}}$.
2. $\mathsf{QMA}_1^{\mathcal{G}_2} = \mathsf{QMA}_1^{\mathcal{G}_4}$, i.e. $\mathbb{Q}$-gates can simulate $\mathbb{Q}(\mathrm{i})$-gates.
3. $\forall k \in \mathbb{Z}_{\geq 0}$: $\mathsf{BQP}_1^{\mathcal{G}} \subseteq \mathsf{BQP}_1^{\mathcal{G}_2}$ for all finite $\mathcal{G} \subseteq \mathrm{U}(2^\ell, \mathbb{Q}(\zeta_{2^k}))$.
4. The *Decision Clique Homology* (CH) problem [KK24] is PSPACE-complete.
   - Decide whether the *clique complex* of a given graph has a hole of a given dimension.
   - [SL23]: Counting number of holes is #P-hard. Note: $\mathsf{P}^{\#\mathsf{P}} \subseteq \mathsf{PSPACE}$.
5. The *Gapped Clique Homology* (GCH) problem [KK24] is $\mathsf{QMA}_1^{\mathcal{G}_2}$-complete.
   - Additional promise on the *combinatorial Laplacian*.
   - [KK24]: GCH is between QMA_1 and QMA.

7. $\forall k \in \mathbb{Z}_{\geq 0} \forall \ell \geq 4$: ℓ-$\mathsf{QSAT}^{\mathbb{Q}(\zeta_{2^k})}$ is $\mathsf{QMA}_1^{\mathcal{G}_{2^k}}$-complete.
   - For $k \geq 3$ (i.e. we have the T-gate), $\ell = 3$ suffices!
8. The *Exact Local Hamiltonian* (ELH) problem in $\mathbb{Q}(\zeta_{2^k})$ is $\mathsf{QMA}_1^{\mathcal{G}_{2^k}}$-complete.
   - Decide whether H has eigenvalue $\lambda = 0$, or all eigenvalues $|\lambda| \geq 1/\mathrm{poly}(n)$.
   - First 2-local QMA_1-complete problem! For $k \geq 3$, 2-ELH is $\mathsf{QMA}_1^{\mathcal{G}_{2^k}}$-complete. [Bra06]: 2-QSAT $\in$ P. (ELH: YES-case not necessarily *frustration-free*)
9. The *Exact Sparse Hamiltonian* (ESH) problem is also QMA_1-complete.
10. The *Exact Separable Sparse Hamiltonian* (ESSH) problem is $\mathsf{QMA}_1(2)$-complete.
    - Decide: $\exists |\psi_1\rangle, |\psi_2\rangle$: $H|\psi_1\rangle|\psi_2\rangle = 0$ or $\forall |\psi_1\rangle, |\psi_2\rangle$: $\| H|\psi_1\rangle|\psi_2\rangle \| \geq 1/\mathrm{poly}(n)$.
    - First complete problem for $\mathsf{QMA}_1(2)$! (SSH is QMA(2)-complete [CS12])

QMA_1 and The Gateset Issue

Definition ($\mathsf{QMA}_1^{\mathcal{G}}$). A promise problem $A = (A_{\mathrm{yes}}, A_{\mathrm{no}})$ is in $\mathsf{QMA}_1^{\mathcal{G}}$ if there exists uniform circuit family $\{Q_x\}$ with gates $\mathcal{G}$ such that:
- (Completeness) $\forall x \in A_{\mathrm{yes}} \exists |\psi\rangle$: $p_{\mathrm{acc}}(Q_x, \psi) = 1$.
- (Soundness) $\forall x \in A_{\mathrm{no}} \forall |\psi\rangle$: $p_{\mathrm{acc}}(Q_x, \psi) \leq 1 - 1/\mathrm{poly}(|x|)$.

- Analogous definitions for BQP_1, $\mathsf{QMA}_1(2)$.

Does QMA_1 have a *universal gateset*?
I.e. does there exist $\mathcal{G}^*$ s.t. $\forall \mathcal{G}$: $\mathsf{QMA}_1^{\mathcal{G}} \subseteq \mathsf{QMA}_1^{\mathcal{G}^*}$?

- Solovay-Kitaev *approximates* any gate with a *universal gateset*. → However, does not preserve **perfect completeness**!

Not a major issue for the *Quantum Satisfiability* (QSAT) problem:

Definition (ℓ-$\mathsf{QSAT}^{\mathbb{K}}$). **Input:** Hamiltonian $H = \sum_{i=1}^m H_i$ on n qubits in field $\mathbb{K}$ with ℓ-local $H_i \succcurlyeq 0$.
- YES. $\lambda_{\min}(H) = 0$.
- NO. $\lambda_{\min}(H) \geq 1/\mathrm{poly}(n)$.

[GN13]: 3-QSAT is $\mathsf{QMA}_1^{\mathcal{G}_8}$-complete for $\mathcal{G}_8 = \{\mathrm{H}, \mathrm{T}, \mathrm{CX}\}$,
- with QSAT restriction: H_i may be projectors in $\mathbb{Z}[\frac{1}{\sqrt{2}}, \mathrm{i}]$ or $UH_iU^\dagger = (\sqrt{1/3}|000\rangle - \sqrt{2/3}|001\rangle)(\sqrt{1/3}\langle 000| - \sqrt{2/3}\langle 001|)$.

[KK24]: *Gapped Clique Homology* is between QMA_1 and QMA.
- Can only embed *rational gates* into clique homology.
- Use *Pythagorean* gateset $\mathcal{G}_{\mathrm{Pyth}} = \{\mathrm{CX}, U_{\mathrm{Pyth}}\}$, $U_{\mathrm{Pyth}} = \frac{1}{5}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$.
- Power of $\mathcal{G}_{\mathrm{Pyth}}$ unclear. Is $\mathsf{QCMA} \subseteq \mathsf{QMA}_1^{\mathcal{G}_{\mathrm{Pyth}}}$? (see [JKNN12])

To show that GCH is QMA_1-complete, we need a gateset $\mathcal{G}$ that is
- **sufficiently powerful** to decide GCH with perfect completeness,
- **rational**, so that it can be embedded into clique homology.

Exact Synthesis

[AGK+24]: An m-qubit unitary can be synthesized *exactly* with the gateset $\mathcal{G}_{2^k}$ (with ancillas) if and only if $U \in \mathrm{U}(2^m, \mathbb{Z}[1/2, \zeta_{2^k}])$.

$$\mathcal{G}_2 = \{\mathrm{X}, \mathrm{CX}, \mathrm{CCX}, \mathrm{H} \otimes \mathrm{H}\}$$
$$\mathcal{G}_4 = \{\mathrm{X}, \mathrm{CX}, \mathrm{CCX}, \zeta_8 \mathrm{H}\}$$
$$\mathcal{G}_{2^k} = \{\mathrm{H}, \mathrm{CX}, \mathrm{T}_{2^k}\}, \quad \mathrm{T}_{2^k} = \begin{pmatrix} 1 & 0 \\ 0 & \zeta_{2^k} \end{pmatrix}$$

This work: Implement all unitaries in $\mathbb{Q}(\zeta_{2^k})$ with $\mathcal{G}_{2^k}$.
- Impossible, even with ancillas. Hence, we need *postselection*!

Linear Combinations of Unitaries

Step 1: Integer state preparation.

Lemma. Let $|\psi\rangle \propto \sum_{i=0}^{d-1} a_i |i\rangle$, $a_i \in \mathbb{Z}$. Can prepare $|\psi\rangle$ with probability $\geq 1/4d$ with gateset $\mathcal{G}_2$ in time $\mathrm{poly}(d, \log A)$ for $A := \sum_i |a_i|$.
Generalizes to $a_i \in \mathbb{Z}[\zeta_{2^k}]$ with gateset $\mathcal{G}_{2^k}$.

- Let $2^n \geq A$. Prepare $|\psi_0\rangle = \mathrm{H}^{\otimes n}|0\rangle \propto \sum_{j=0}^{2^n-1} |j\rangle_{\mathcal{B}}$.
- Perform measurement to project onto $|j\rangle, j < A$: $|\psi_1\rangle \propto \sum_{j=0}^{A-1} |j\rangle_{\mathcal{B}}$.
- Group the $|j\rangle$: $|\psi_2\rangle \propto \sum_{i=0}^{d-1} |i\rangle_{\mathcal{A}} \sum_{j=0}^{a_i-1} |j + k_i\rangle_{\mathcal{B}}$ for $k_i = \sum_{j=0}^{i-1} |a_j|$.
- Subtract k_i: $|\psi_3\rangle \propto \sum_{i=0}^{d-1} |i\rangle_{\mathcal{A}} \sum_{j=0}^{a_i-1} |j\rangle_{\mathcal{B}}$.
- Project $\mathcal{B}$ onto $|+\rangle^{\otimes m}$: $|\psi_3\rangle \propto \sum_{i=0}^{d-1} |a_i| \cdot |i\rangle_{\mathcal{A}}$
- Flip sign of $|i\rangle$ if $a_i < 0$ to get $|\psi\rangle$.

Step 2: Gate application. Use LCU [CW12].

Lemma. Let $U \in \mathrm{U}(2^n, \mathbb{Q}(\mathrm{i}))$ with $sU \in \mathbb{Z}[\mathrm{i}]$ for $s^2 \in \mathbb{N}$. U can be implemented with success probabily $2^{-\Omega(n)}$ in time $\mathrm{poly}(2^n, \log s)$ using $\mathcal{G}_4$.
Generalizes to $U \in \mathrm{U}(2^n, \mathbb{Q}(\zeta_{2^k}))$ with gateset $\mathcal{G}_{2^k}$ for $k > 2$ with success probability $1/\mathrm{poly}(2^n, s)$.

Decompose U into *Pauli basis*: $U = \sum_{x \in \{0,1\}^{2n}} a_x P_x$, $a_x \in \mathbb{Q}(\mathrm{i})$ where
- $P_x = \bigotimes_{j=1}^n \sigma_{x_{2j-1}x_{2j}}$, and $\sigma_{00} = \mathrm{I}$, $\sigma_{01} = \mathrm{X}$, $\sigma_{10} = \mathrm{Y}$, $\sigma_{11} = \mathrm{Z}$.
- Prepare $|U\rangle_{\mathcal{A}} = \sum_{x \in \{0,1\}^{2n}} a_x |x\rangle$.
- On input $|\phi\rangle_{\mathcal{B}}$, apply P_x conditioned on x:

$$|U\rangle_{\mathcal{A}}|\phi\rangle_{\mathcal{B}} = \sum_{x \in \{0,1\}^{2n}} a_x |x\rangle_{\mathcal{A}} |\phi\rangle_{\mathcal{B}} \mapsto \sum_{x \in \{0,1\}^{2n}} a_x |x\rangle_{\mathcal{A}} \otimes P_x |\phi\rangle_{\mathcal{B}} =: |\psi'\rangle$$

- Measure $\mathcal{A}$ in X-basis. Outcome $|y_{\mathrm{H}}\rangle := \mathrm{H}^{\otimes 2n}|y\rangle$ for $y \in \{0,1\}^{2n}$.
- Succeed if $y = 0^{2n}$: $(\langle +|_{\mathcal{A}}^{2n} \otimes I_{\mathcal{B}})|\psi'\rangle \propto \sum_{x \in \{0,1\}^{2n}} a_x P_x |\phi\rangle_{\mathcal{B}} = U|\phi\rangle_{\mathcal{B}}$
- For $y \neq 0^{2n}$, a different unitary $P_{\bar{y}} U P_{\bar{y}}$ is applied.
- $\Pr[y] = \| (\langle y_{\mathrm{H}}|_{\mathcal{A}}^{2n} \otimes I_{\mathcal{B}})|\psi'\rangle \|^2 = 1/4^n$.

Theorem. $\mathsf{QMA}_1^{\mathcal{G}} \subseteq \mathsf{QMA}_1^{\mathcal{G}_{2^k}}$ for finite $\mathcal{G}$ in $\mathbb{Q}(\zeta_{2^k})$.

- Use *Oblivious amplitude amplification* [BCC+14] to boost success probability of gate application.

Simulating Cyclotomic Gates with Integer Gates

[McK13]: Simulate $\mathbb{R}$ with $\mathbb{C}$. Here: simulate $\mathbb{Q}(\zeta_{2^k}) =: \mathbb{K}$ with $\mathbb{Q}$.

Encoding: Let $d = 2^{k-1}$ be the degree of ζ_n, $n = 2^k$.
- $a = \sum_{i=0}^{d-1} a_i \zeta_n^i \in \mathbb{Q}(\zeta_n) \quad \mapsto \quad |v(a)\rangle = \sum_{i=0}^{d-1} a_i |i\rangle$ with all $a_i \in \mathbb{Q}$
- $|\psi\rangle = \sum_{i=0}^{N-2} a_i |i\rangle \in \mathbb{Q}(\zeta_n)^N \quad \mapsto \quad |v(\psi)\rangle \propto \sum_{i=0}^{d-1} |i\rangle_a |v(a_i)\rangle$.

Lemma. $\exists$ *group homomorphism* $\Psi : \mathrm{U}(N, \mathbb{K}) \to \mathrm{O}(dN, \mathbb{Q})$, s.t. $\Psi(U)|v(\psi)\rangle = |v(U|\psi\rangle)\rangle$ for all $|\psi\rangle \in \mathbb{K}^N$.

Ψ acts entry-wise: For $a \in \mathbb{K}$, let $M_a := \Psi(a)$, s.t. $M_a|v(b)\rangle = |v(ab)\rangle$.
- Only for $n = 2^k$, we have $M_a^T = M_{\bar{a}}$ as $\overline{\zeta_n} = \zeta_n^{-1}$ with

$$M_{\zeta_n} = \begin{pmatrix} 0 & & & -1 \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ & & 1 & 0 \end{pmatrix}, \quad M_{\zeta_n}^{-1} = M_{\zeta_n}^T = \begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ -1 & & & 0 \end{pmatrix}.$$

$$\sqrt{\mathrm{H}} = \left(\begin{array}{c|c} -\frac{1}{2}\zeta_8^3 + \frac{1}{2}\zeta_8^2 + \frac{1}{2} & -\frac{1}{2}\zeta_8^3 \\ \hline -\frac{1}{2}\zeta_8^3 & \frac{1}{2}\zeta_8^3 + \frac{1}{2}\zeta_8^2 + \frac{1}{2} \end{array}\right) \overset{\Psi}{\longmapsto} \left(\begin{array}{cccc|cccc} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ \hline 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \end{array}\right)$$

Theorem. $\mathsf{QMA}_1^{\mathcal{G}_4} = \mathsf{QMA}_1^{\mathcal{G}_2}$. $\mathsf{BQP}^{\mathcal{G}} \subseteq \mathsf{BQP}^{\mathcal{G}_2}$ for finite $\mathcal{G}$ in $\mathbb{Q}(\zeta_{2^k})$.

- For $k \geq 3$, prover could make 0 from $1, \zeta_n, \zeta_n^2$ to send "0-proof".

Sparse Hamiltonians

Lemma. Let $H \in \mathbb{Z}[\zeta_{2^k}]^{2^n \times 2^n}$ be an n-qubit d-*sparse* Hamiltonian with L-bit entries. Can decompose $H = \sum_{i=1}^m \sum_{l=0}^{L-1} 2^{l-1} U_{i,l}$ with $m = O(2^k d^2)$ and $U_{i,l}$ efficiently implementable with $\mathcal{G}_{2^k}$.

Proof idea. [BCC+14]: Can write $H = \sum_{i=1}^{d^2} H_i$ with 1-sparse H_i. [KL21; BCC+14] decompose 1-sparse Hamiltonians into sums of 1-sparse unitaries. Modify construction for cyclotomic setting.

Theorem. $\mathsf{ESH}^{\mathbb{Q}(\zeta_{2^k})} \in \mathsf{QMA}_1^{\mathcal{G}_{2^k}}$.

- Prover sends $|\phi\rangle \in \mathrm{Ker}(H)$. Try to apply H to $|\phi\rangle_{\mathcal{B}}$.
- Prepare $|H\rangle_{\mathcal{A}} \propto \sum_{i=1}^m \sum_{l=0}^{L-1} 2^{l-1} |i, l\rangle$. Conditionally apply $(U_{i,l})_{\mathcal{B}}$.
- Measure $\mathcal{A}$ in X-basis and *reject* if $H|\phi\rangle$ was successfully prepared.
- $\Pr[\text{reject}] \propto \|H|\phi\rangle\|^2$. Never reject $H|\phi\rangle = 0$!

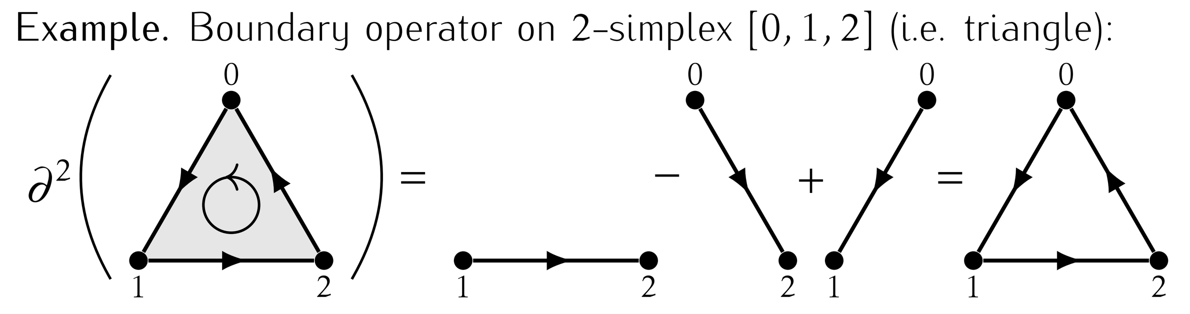
Theorem. $\mathsf{ESSH}^{\mathbb{Q}(\zeta_{2^k})}$ is $\mathsf{QMA}_1^{\mathcal{G}_{2^k}}(2)$-complete.

- Hardness by [CS12], containment above.

Clique Homology

Simplicial complex. Collection of simplices $\mathcal{K} = \mathcal{K}^0 \cup \mathcal{K}^1 \cup \cdots$, s.t. $\forall \sigma \in \mathcal{K}^k$: $|\sigma| = k + 1$ and $\forall \tau \subset \sigma$: $\tau \in \mathcal{K}$.
- Let $\mathcal{C}^k(\mathcal{K}) = \mathrm{Span}\{|\sigma\rangle \mid \sigma \in \mathcal{K}^k\}$, with canonical ordering $\sigma = [v_0, \dots, v_k]$. Identify $|[\pi(v_0), \dots, \pi(v_k)]\rangle = (-1)^{\mathrm{sgn}(\pi)} |[v_0, \dots, v_k]\rangle$ for all $\pi \in S_{k+1}$.
- *Boundary operator* $\partial^k |[v_0 \dots v_k]\rangle = \sum_{i=0}^k (-1)^i |[v_0 \dots \widehat{v}_i \dots v_k]\rangle$.
- *Homology group* $H^k = \mathrm{Ker}\, \partial^k / \mathrm{Im}\, \partial^{k+1}$, i.e. a *cycle* $\partial^k |\psi\rangle = 0$ is a *hole* if it is not a *boundary* ($|\psi\rangle \in \mathrm{Im}(\partial^{k+1})$).
- *Coboundary operator* $d^k = (\partial^k)^\dagger$ via inner product $\langle \sigma | \tau \rangle = \delta_{\sigma,\tau}$.
- *Laplacian* $\Delta^k = d^{k-1}\partial^k + \partial^{k+1} d^k$ with $\mathrm{Ker}\, \Delta^k \cong H^k$.

Example. Boundary operator on 2-simplex $[0, 1, 2]$ (i.e. triangle):



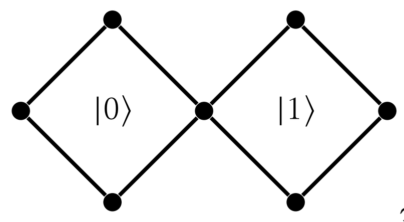
$\partial^1(|01\rangle + |12\rangle + |20\rangle) = (|1\rangle - |0\rangle) + (|2\rangle - |1\rangle) + (|2\rangle - |0\rangle) = 0$

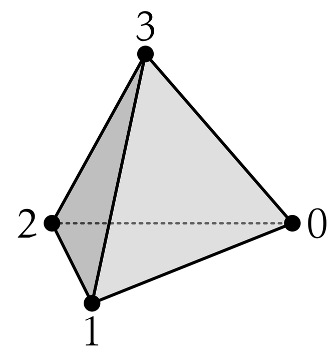
$\Rightarrow \partial^k \circ \partial^{k+1} = 0$.

Definition (Decision Clique Homology (CH)). **Input:** Graph G and $k \in \mathbb{N}$. Decide if simplicial complex of cliques of G has $H^k = 0$.

Examples of *clique complexes*:

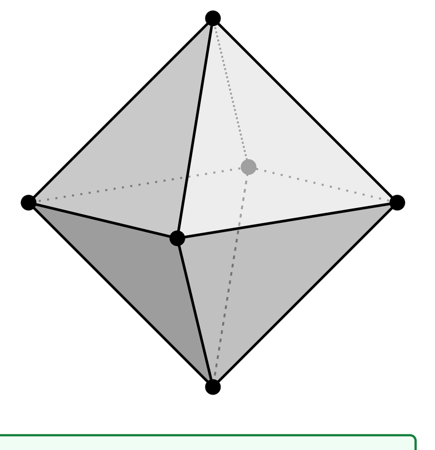
Qubit gadget of [KK24] has two 1-holes.





3-simplex $[0, 1, 2, 3]$ has no holes.

Octahedron is S^2 and has a 2-hole.



Theorem. CH is PSPACE-complete.

- CH $\in$ PSPACE by computing the rank of Laplacian Δ^k [Chi85].
- [Li22]: $\mathsf{QMA}_{1,1-1/\exp}^{\mathcal{G}_2} = \mathsf{PSPACE} \Rightarrow$ 4-$\mathsf{QSAT}^{\mathbb{Q}}$ is PSPACE-complete.
- Adapt clique homology gadgets of [KK24] to our 4-$\mathsf{QSAT}^{\mathbb{Q}}$ Hamiltonian. → Verify new gadgets computationally.

Definition (Gapped Clique Homology (GCH) [KK24]).
- **Input.** Graph $G = (V, E)$ with weights $w : V \to \mathbb{Q}$. For each clique/simplex $\sigma = [v_0, \dots, v_k]$, let $w(\sigma) = \prod_{i=0}^k w(v_i)$. Redefine inner product $\langle \sigma | \tau \rangle = w(\sigma)^2 \delta_{\sigma,\tau}$.
- YES. $\lambda_{\min}(\Delta^k) = 0$.
- NO. $\lambda_{\min}(\Delta^k) \geq 1/\mathrm{poly}(n)$.

Theorem. GCH is $\mathsf{QMA}_1^{\mathcal{G}_2}$-complete.

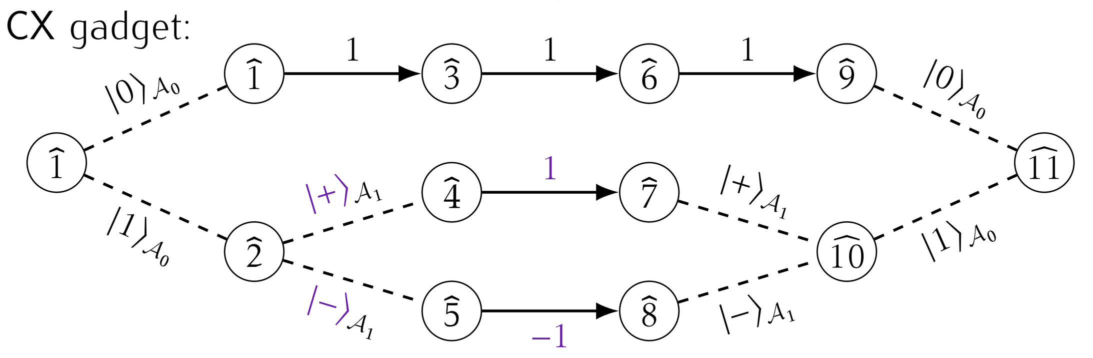
- Containment follows by reduction to SSH.

2-local Hamiltonian

Theorem. 2-$\mathsf{ELH}^{\mathbb{Q}(\zeta_{2^k})}$ is $\mathsf{QMA}_1^{\mathcal{G}_{2^k}}$-complete for all $k \geq 3$.

Recall 2-QSAT $\in$ P. Also cannot use 2-LH of [KKR05] as $H|\psi_{\mathrm{hist}}\rangle \neq 0$.

Idea. Build frustrated Hamiltonian that "behaves like QSAT".
- $H_{\mathrm{clock}} = 4T \sum_{1 \leq i < j \leq T+1} |11\rangle\langle 11|_{i,j} + \sum_{t=1}^{T+1} (|0\rangle\langle 0| - T|1\rangle\langle 1|)_t \succcurlyeq 0$, $\mathrm{Ker}(H_{\mathrm{clock}}) = \mathrm{Span}\{|\hat{t}\rangle \mid t \in \{0, \dots, T\}\}$, $|\hat{t}\rangle := |0^t\, 1\, 0^{T-t}\rangle$
- Implement $\mathrm{X} = |+\rangle\langle +| - |-\rangle\langle -|$ by splitting computation on X-basis. CX gadget:



- By splitting along eigenbasis, a 1-local gate becomes 0-local!

Open Problems

- QMA_1 vs. QMA: Classical oracle separation?
- Can we extend the $\mathsf{BQP}_1^{\mathcal{G}_{2^k}} = \mathsf{BQP}_1^{\mathcal{G}_2}$ results to QMA_1?
- Apply our techniques to obtain more QMA_1-completeness results.
- $k < 3$: $\mathsf{QMA}_1^{\mathcal{G}_{2^k}}$, completeness of 3-QSAT and 2-ELH remain open.

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